Reusable Non-Interactive Secure Computation

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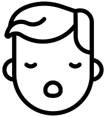
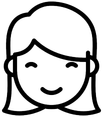
Vinod Vaikuntanathan	(MIT)

Aug 22, 2019

Non-Interactive Secure Computation (NISC)

S 

y

 R

x

Goal: receiver gets $f(x, y)$ for a public function f.

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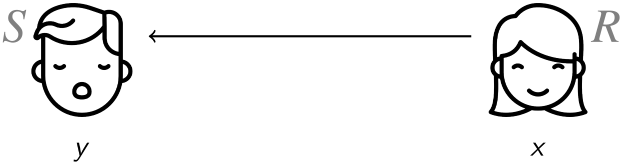

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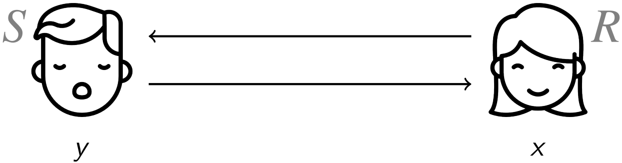

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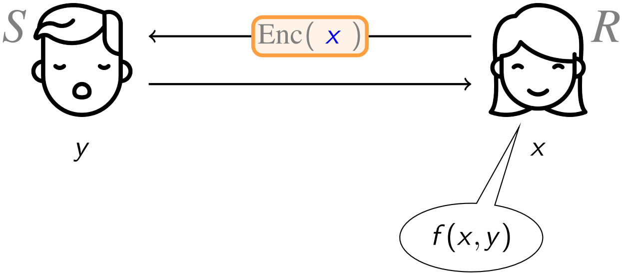

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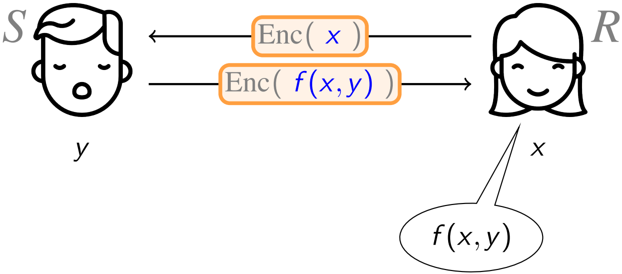

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Garbled Circuit + OT $\Longrightarrow$ Semi-honest NISC [Kilian'88]

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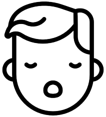

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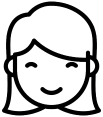

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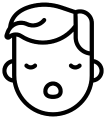

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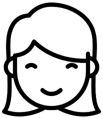

x

$\tilde{C}$ and tags

$w_{1,0}$	$w_{1,1}$
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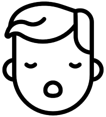

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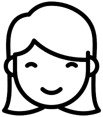

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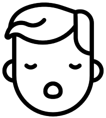

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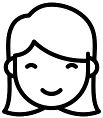

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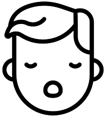

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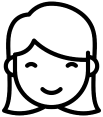

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$\tilde{C}$ and $(w_{i,x_i})_{i=1}^{n}$ reveals $f(x,y)$
and nothing else computationally.

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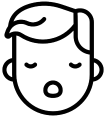
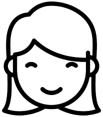


y

$\xrightarrow{\quad \tilde{C} \quad}$

R



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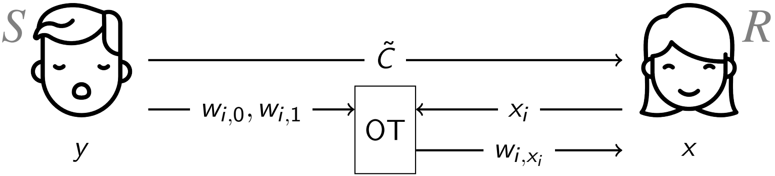

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$$x = \begin{array}{|c|}\hline 1 \\ \hline 0 \\ \hline 0 \\ \hline 1 \\ \hline \vdots \\ \hline 1 \\ \hline \end{array}$$

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NISC in OT-hybrid model

Advantages

- OT realization from various models/assumptions
- Efficiency
- Malicious Security [Ishai-Kushilevitz-Ostrovsky-Prabhakaran-Sahai'88]
  - Information-theoretical NISC for $\mathbf{NC}^0$ in OT-hybrid.
  - NISC in OT-hybrid using black-box PRG.

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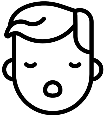
S 

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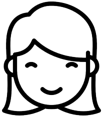 R

x

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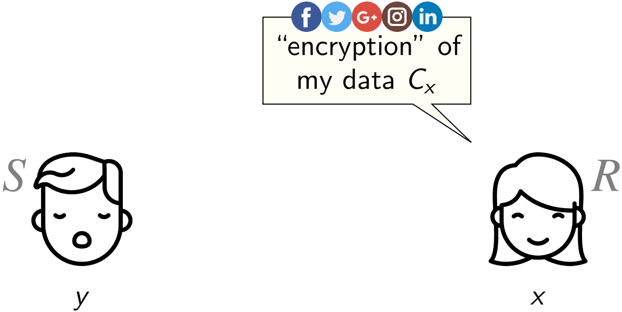

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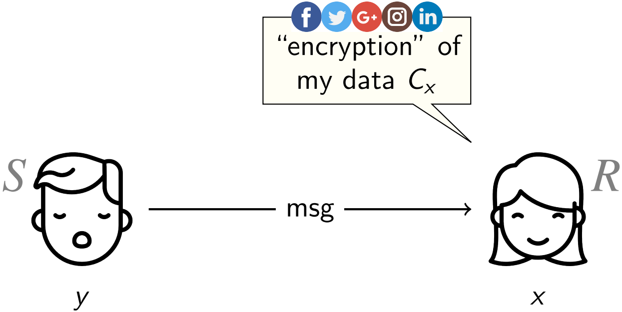

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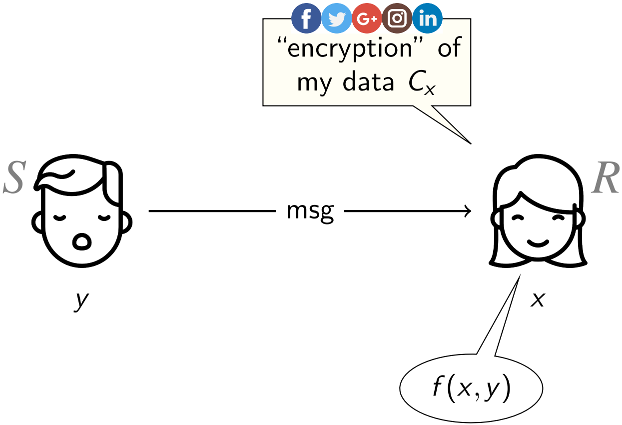

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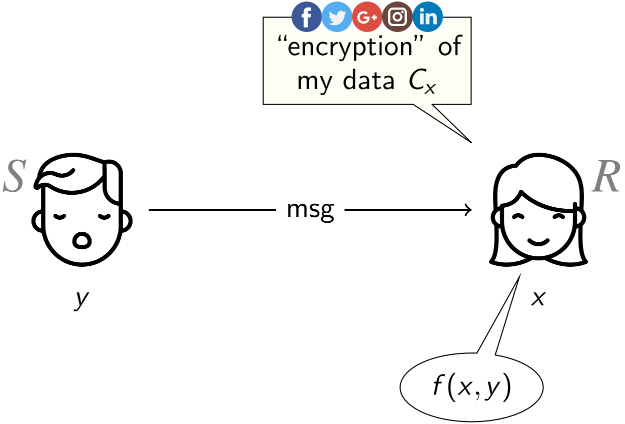

Reusability: Safe for receiver to reuse first msg and randomness

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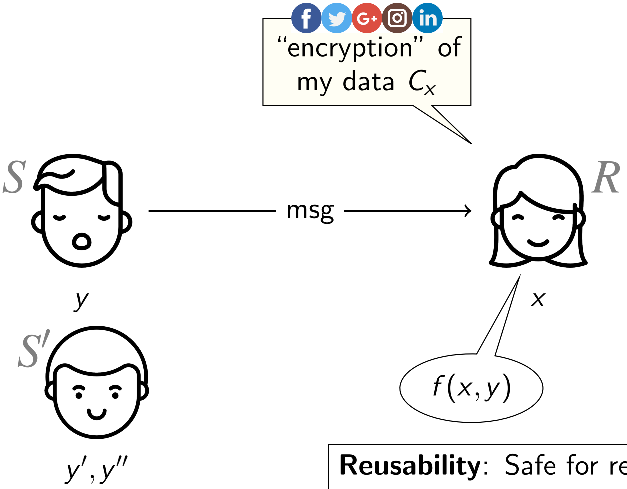

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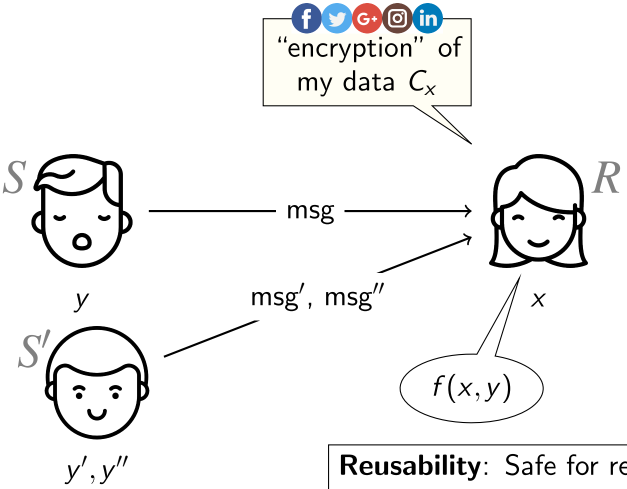

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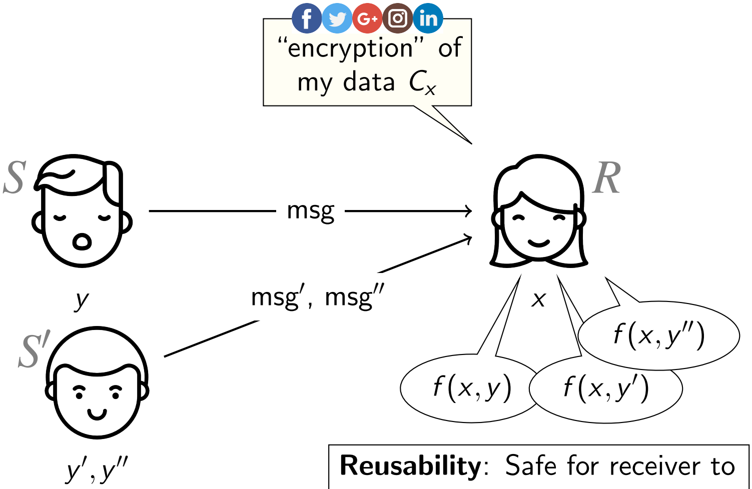

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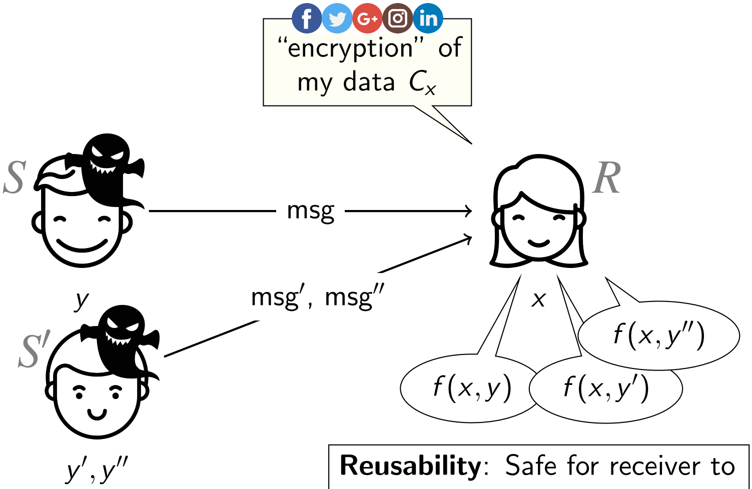

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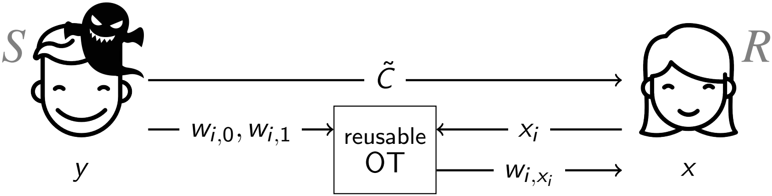

$\tilde{C}$ and tags

$w_{1,0}$	$w_{1,1}$
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$w_{3,0}$	$w_{3,1}$
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$\vdots$	
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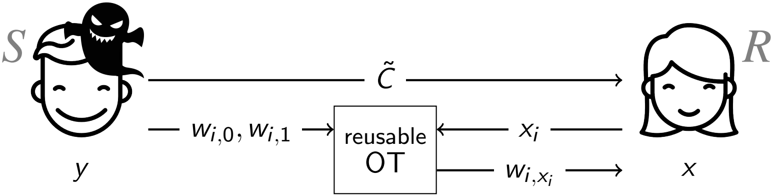
$x =$

1
0
0
1
$\vdots$
1

NISC in OT-hybrid model



$\tilde{C}$ and tags

$w_{1,0}$	mess
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$\vdots$	
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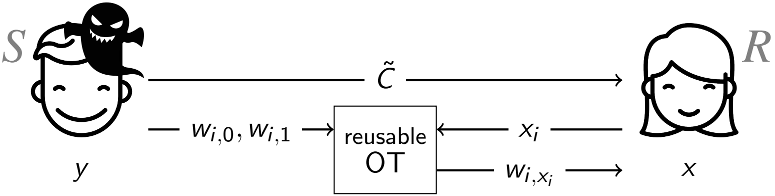

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$\vdots$	
$w_{n,0}$	$w_{n,1}$

$$x = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Replacing $w_{1,1}$ changes 's behaviour $\implies x[1] = 1$
thus **NO security** against malicious sender.

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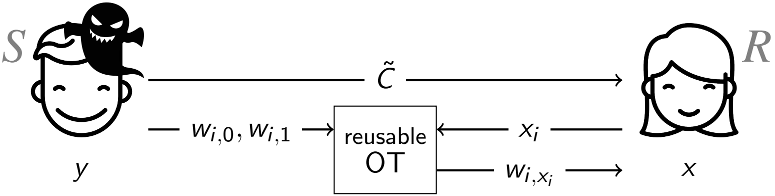

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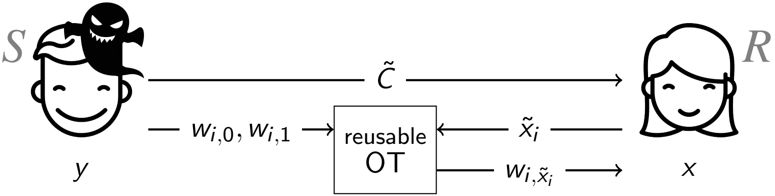
NISC in OT-hybrid model + one-shot UC-security [IKOPS'11]



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let OT input be encoding $\tilde{x} =$

1
0
0
1
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1

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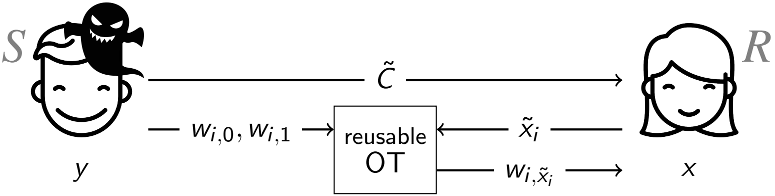

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A few bits of $\tilde{x}$ leaks no information about x.

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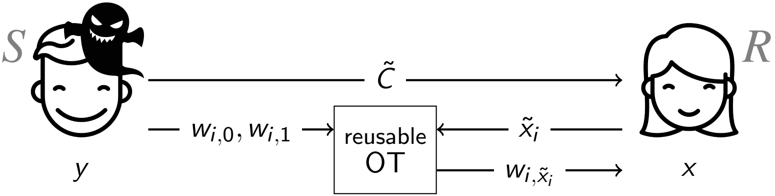

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let OT input be encoding $\tilde{x} =$

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Repeat the attack to learn the whole encoding $\tilde{x}$
thus **NO reusable security** against malicious sender.

Our Results

Impossible to patch the protocol against malicious adversaries in reusable settings, as we show...

Theorem 1

There is no information-theoretic reusable NISC in rOT-hybrid model.

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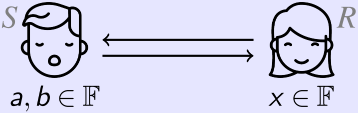
Expansive alternative:
Semi-honest NISC $+$ reusable NIZK $\implies$ reusable NISC.

Our Results (continue)

NEW primitive: Oblivious linear function evaluation (OLE)



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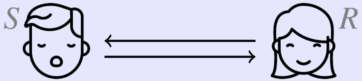

$a, b \in \mathbb{F}$ $\qquad$ $x \in \mathbb{F}$

get $ax + b \in \mathbb{F}$

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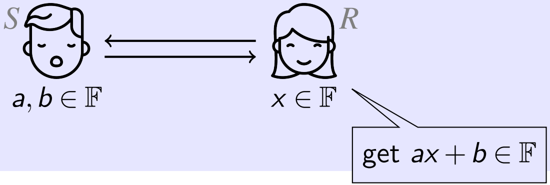

Theorem 2

An information-theoretical UC-secure reusable NISC protocol in rOLE-hybrid model.

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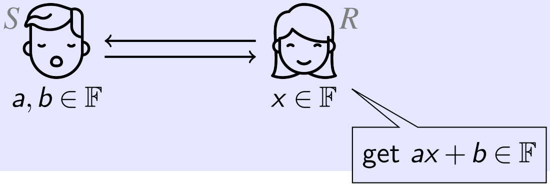

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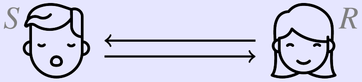

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Degenerate into OT if $|\mathbb{F}| = 2$.

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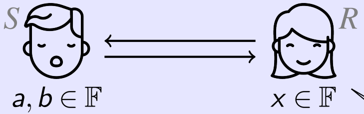

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Security loss $\approx \frac{1}{|\mathbb{F}|}$

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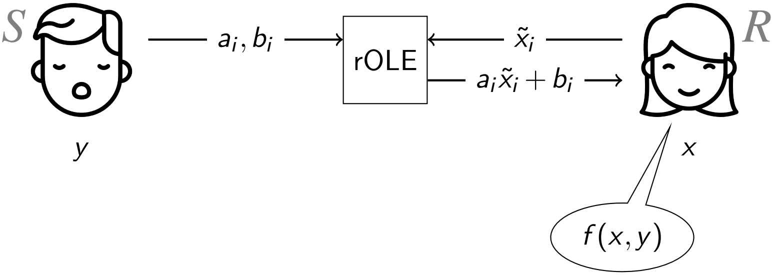
An UC-secure 2-msg reusable OLE protocol in the CRS setting, under Paillier assumption.

How to Lift One-shot Security to Reusability



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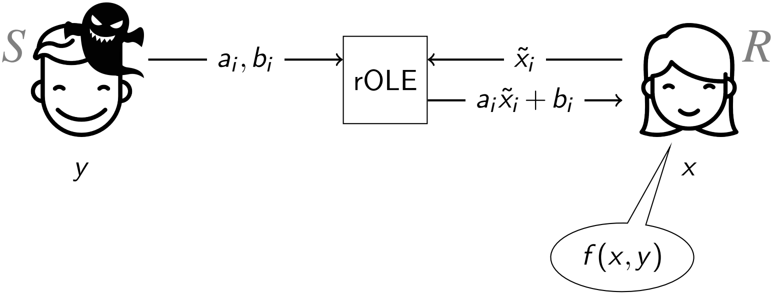

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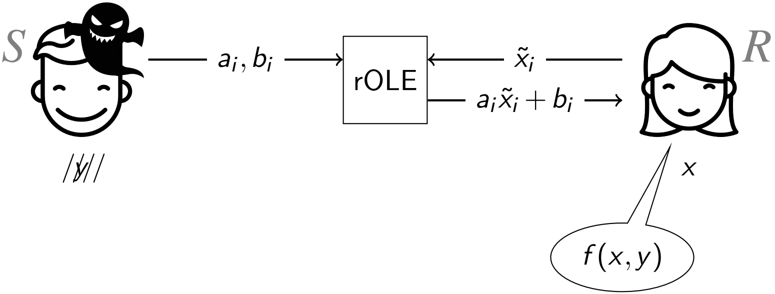

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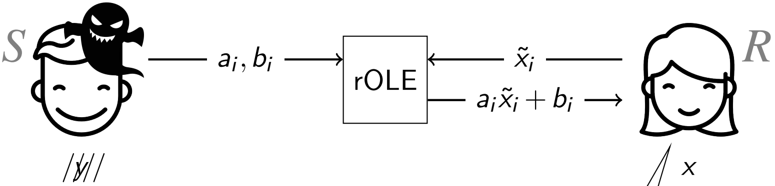

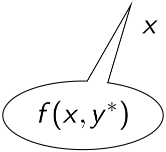

- **UC-security**: $\exists$ an efficient simulator $\mathscr{S}$
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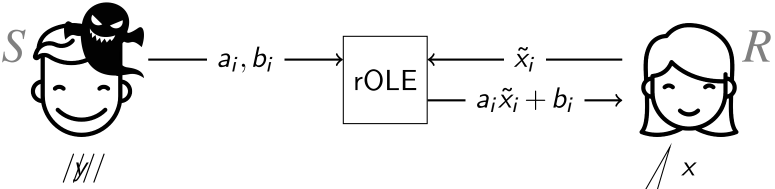

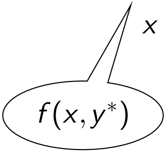

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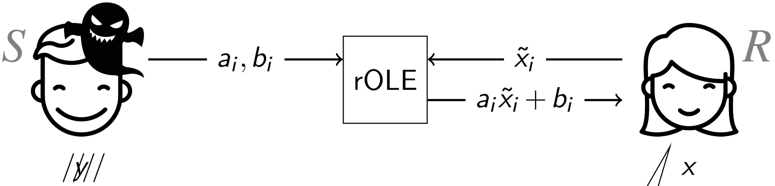

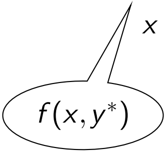

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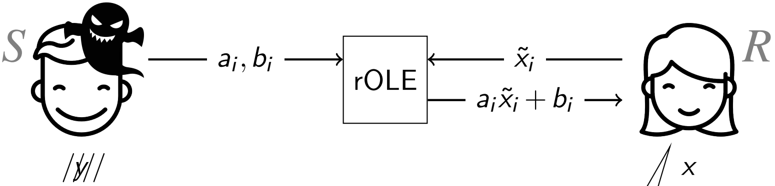

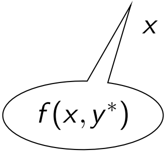

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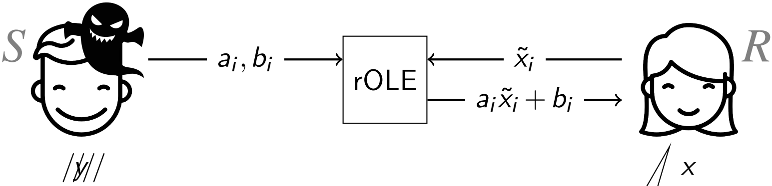

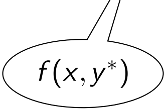

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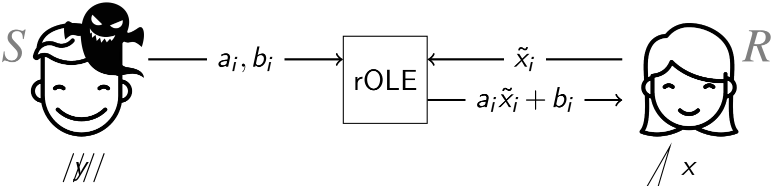

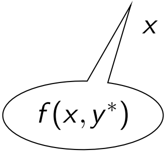

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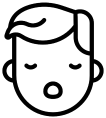
- **"Strong" UC-security $\implies$ Reusability**
  The simulator is deterministic

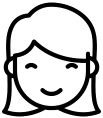
Overview: rNISC in rOLE-hybrid model

S



$y \in \mathbb{F}^n$

R



$x \in \mathbb{F}^n$

- Assume f is an arithmetic $\mathbf{NC}^1$ circuit or an arithmetic branching program over $\mathbb{F}$
- [IK'02,AIK'14] encode $y \mapsto (A, b)$ s.t. $Ax + b$ reveals $f(x, y)$ and nothing else
- **Against malicious sender:** detect if (A, b) is honestly generated, i.e. satisfies some simple arithmetic constraints

$$\text{Certified rOLE} \rightarrow \begin{cases} Ax + b, & \text{if } (A, b) \text{ satisfies constraints} \\ \perp, & \text{otherwise} \end{cases}$$

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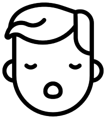

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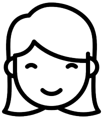

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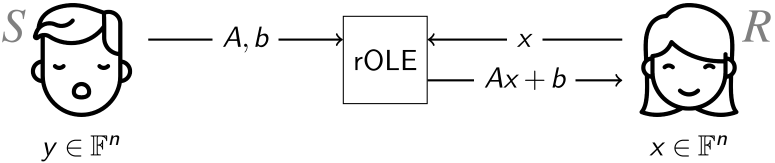

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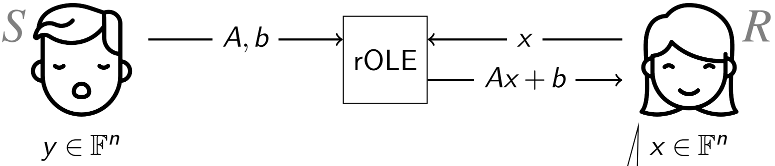

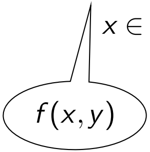

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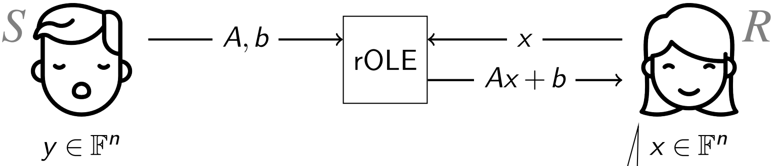

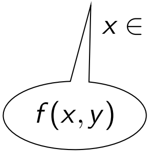

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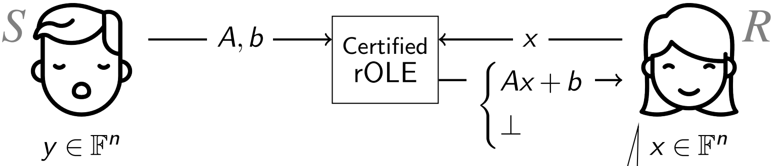

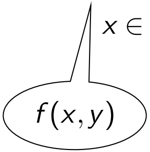

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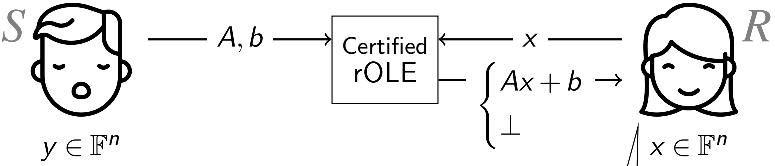

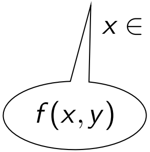

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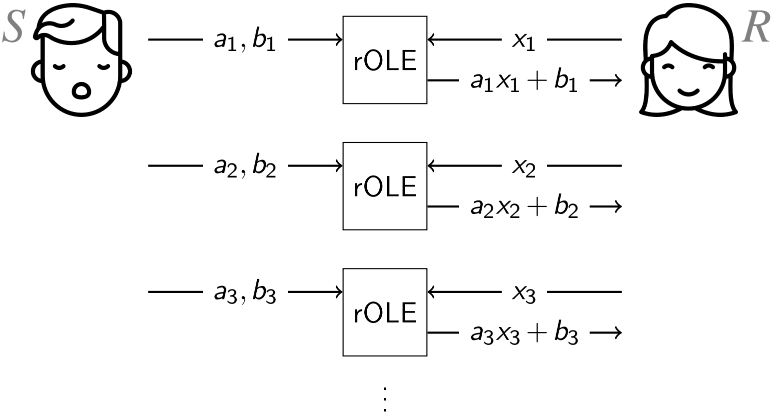

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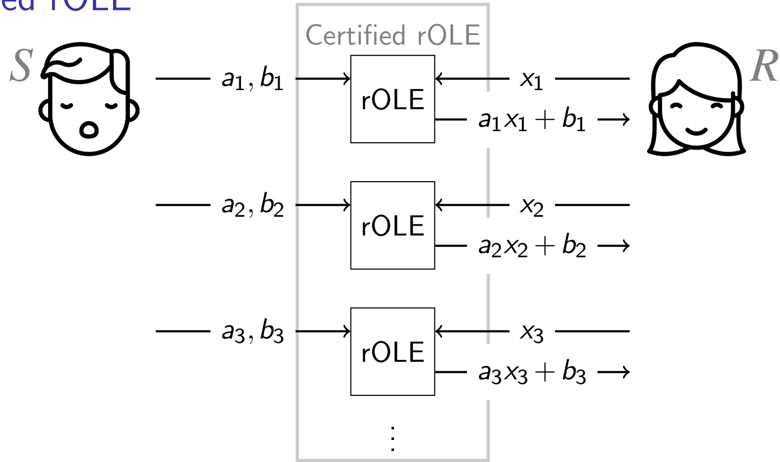

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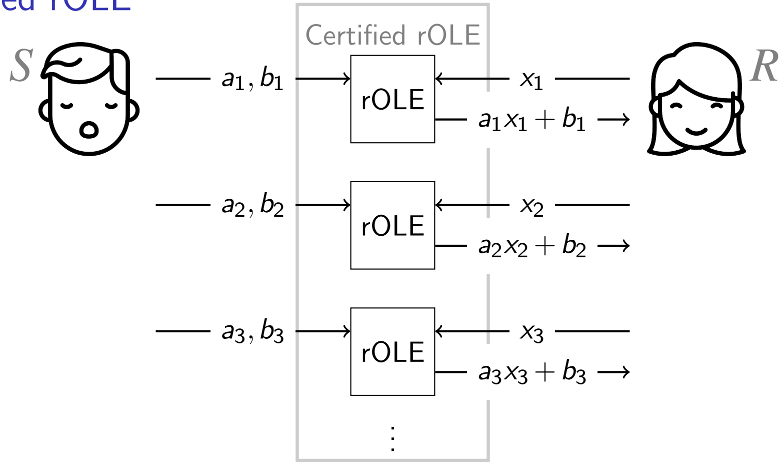

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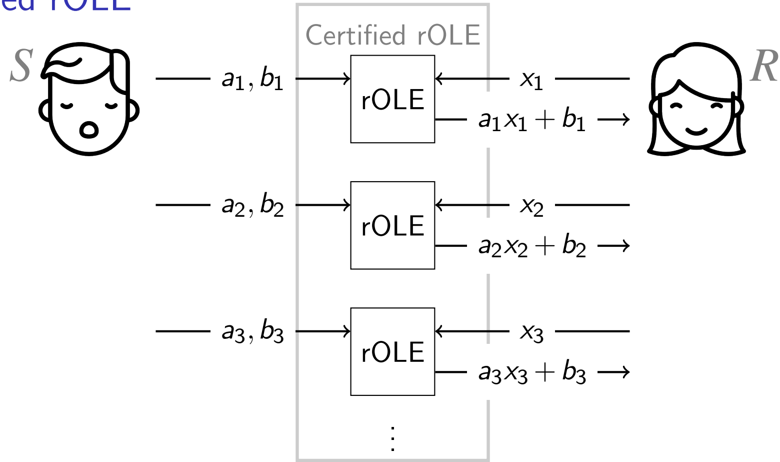

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- Side product: reusable DV-NIZK in rOLE-hybrid model.

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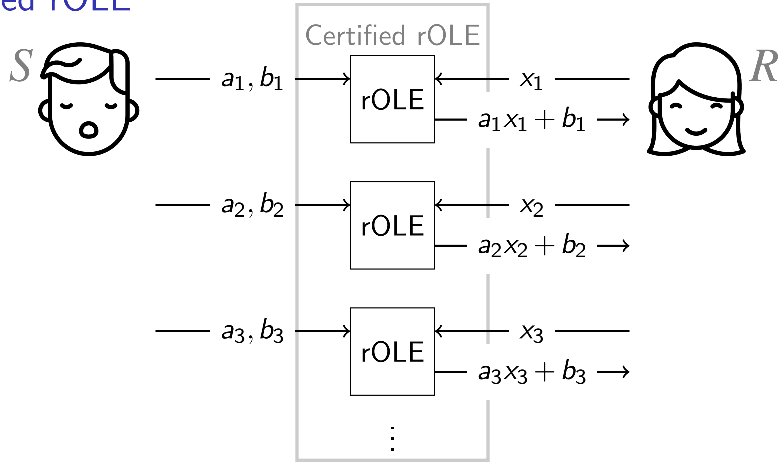

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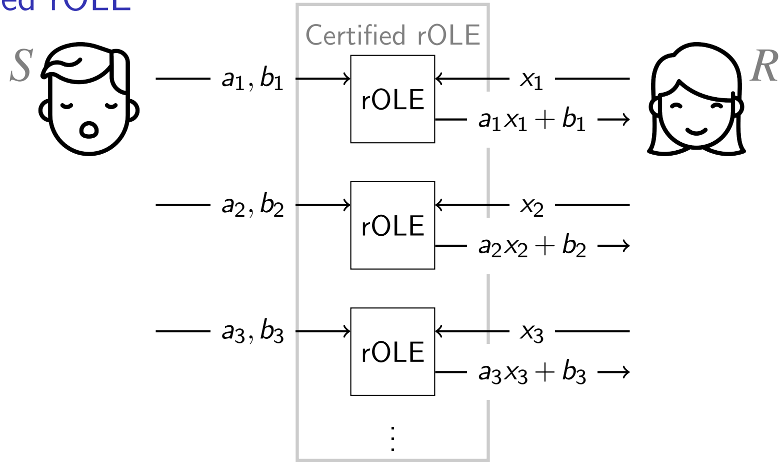

- Sender can prove $(a_1, b_1, a_2, b_2, \ldots)$ satisfies ~~arithmetic constraints~~ $a_i = a_j$ for some (i, j)

  for general constraints $\rightarrow$ see eprint
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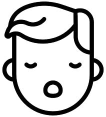
Certified rOLE

S 

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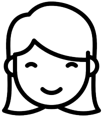 R

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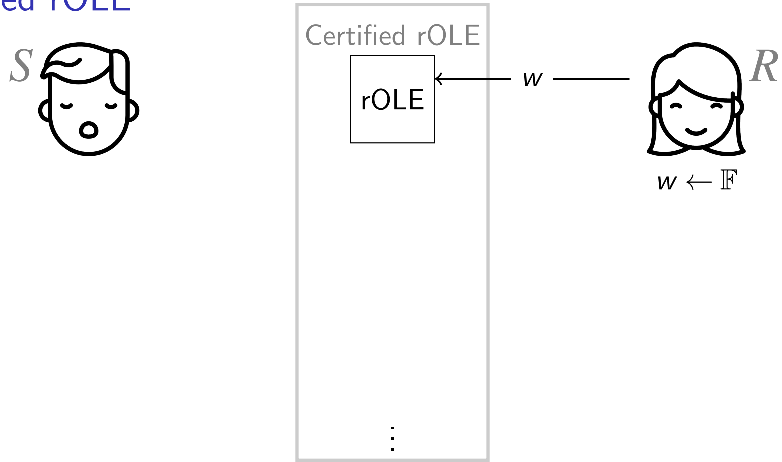

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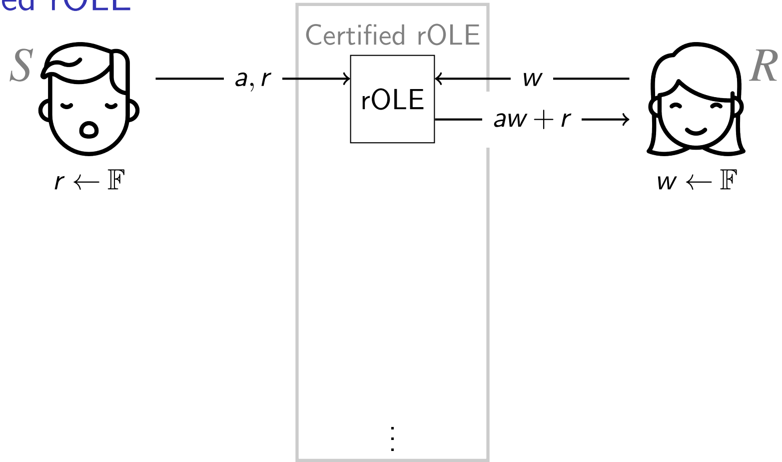

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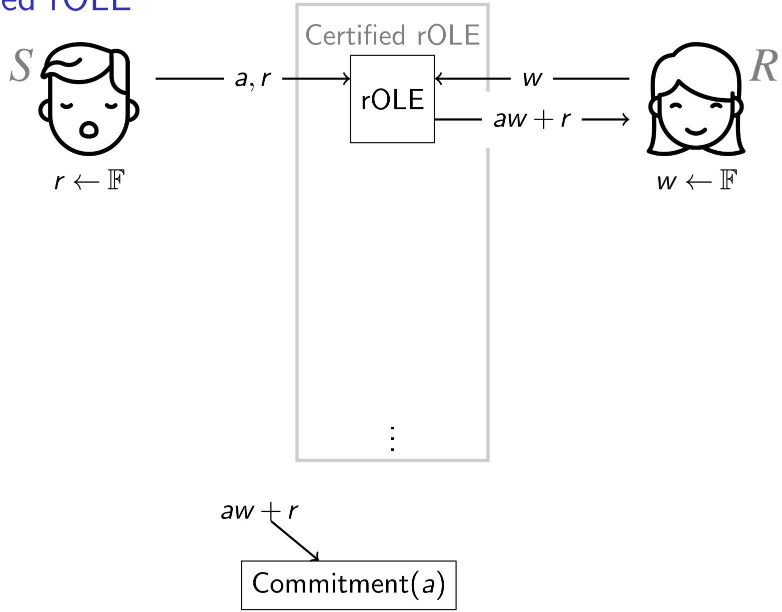

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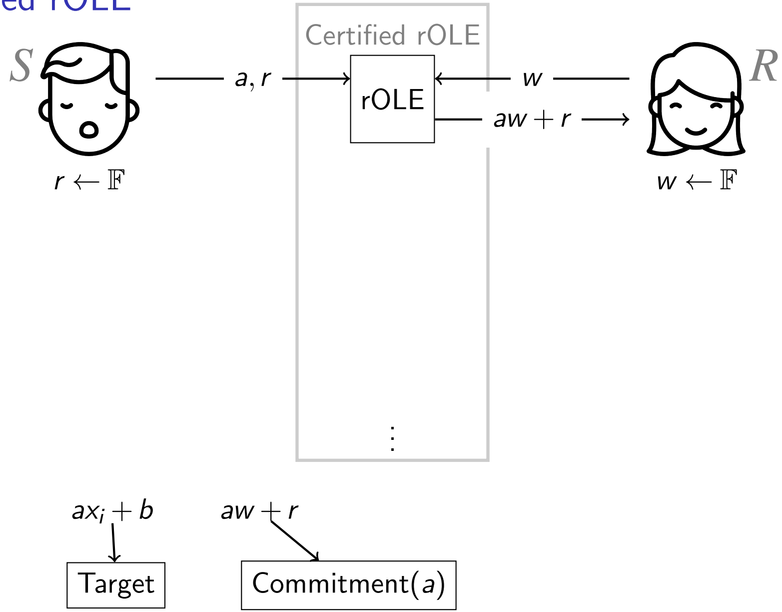

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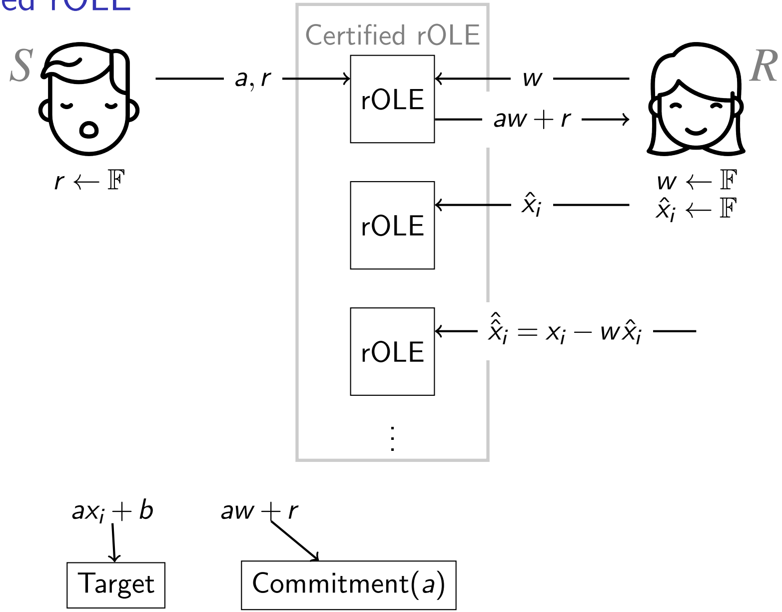

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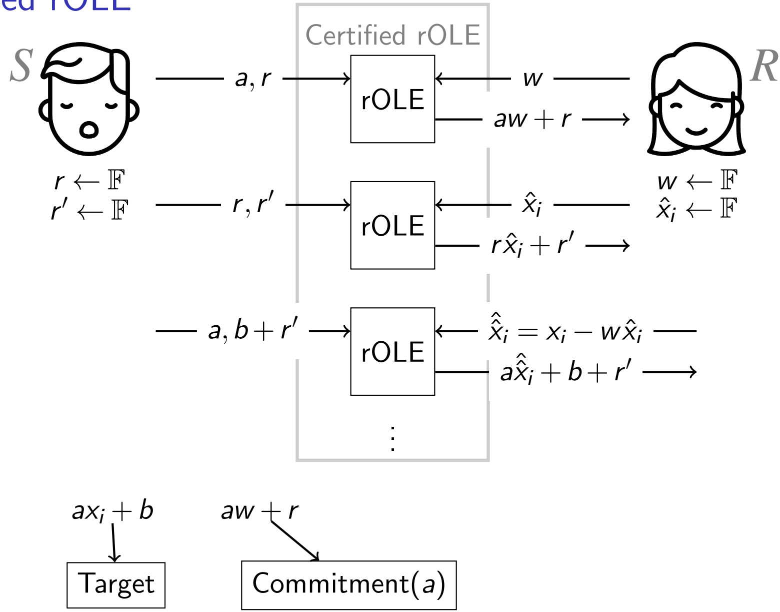

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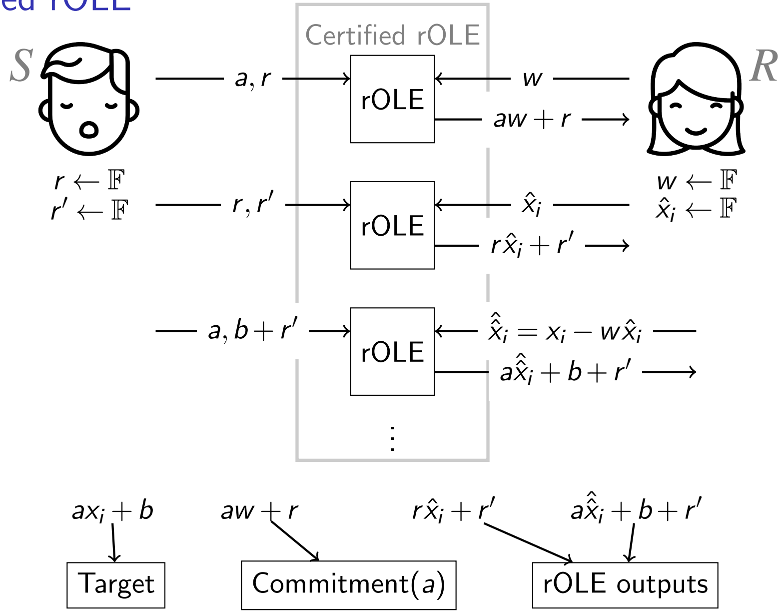

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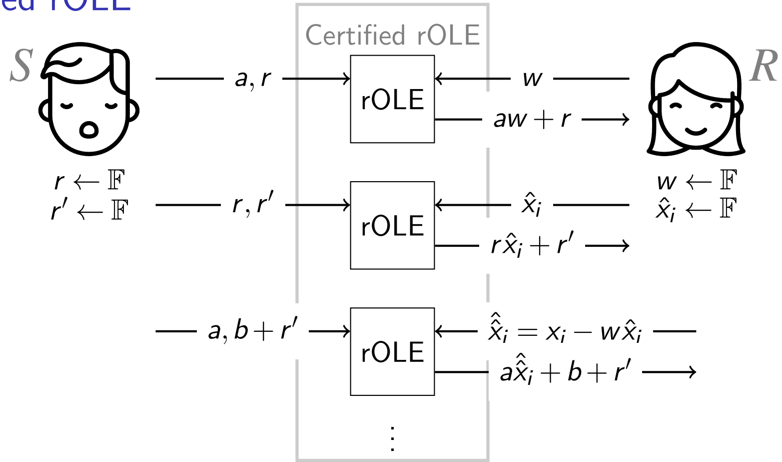


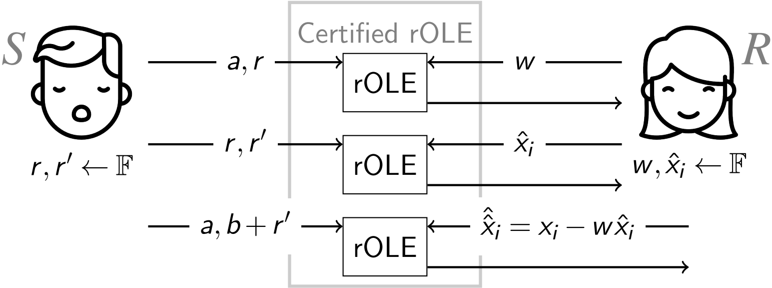
$$ax_i + b \;=\; (aw + r)\cdot \hat{x}_i \;-\; (r\hat{x}_i + r') \;+\; (a\hat{\hat{x}}_i + b + r')$$

Target

Commitment(a)

rOLE outputs

Certified rOLE



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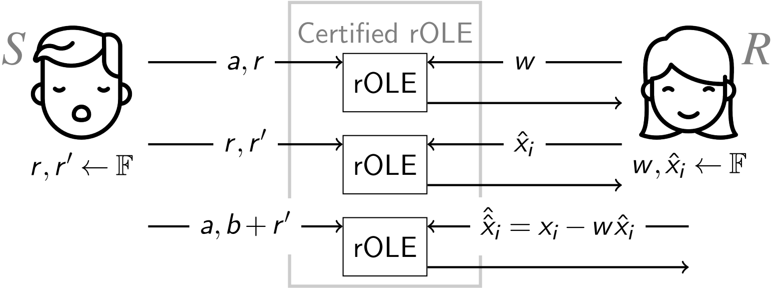

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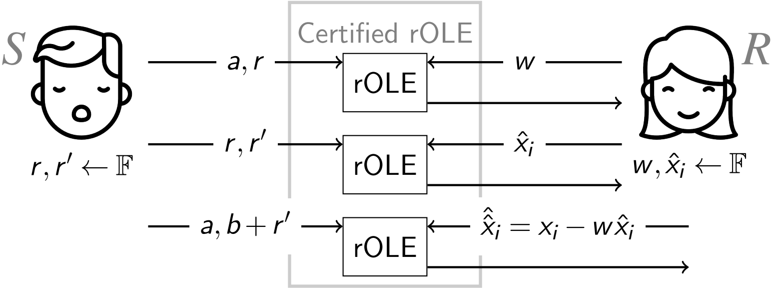

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- Correctness: Above equation.
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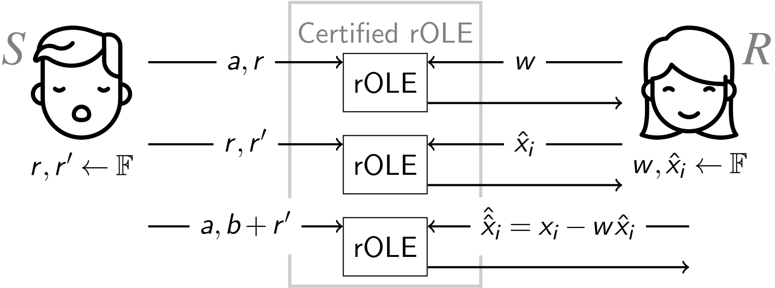

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Target

Commitment(a)

rOLE outputs

- Correctness: Above equation.
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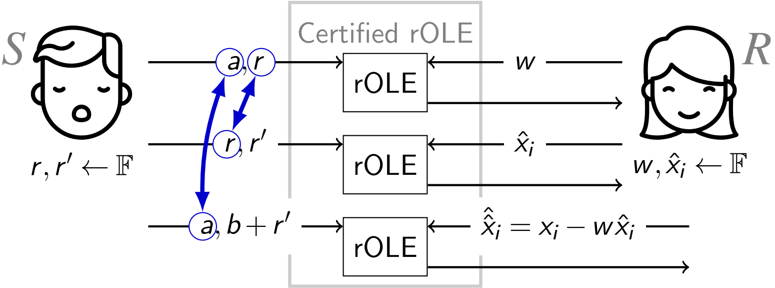
- "Strong" UC-secure against Sender:

Certified rOLE



$$\underbrace{ax_i + b}_{\boxed{\text{Target}}} \;=\; \underbrace{(aw + r)}_{\boxed{\text{Commitment}(a)}} \cdot \hat{x}_i \;-\; \underbrace{(r\hat{x}_i + r') \;+\; e(a\hat{\hat{x}}_i + b + r')}_{\boxed{\text{rOLE outputs}}}$$

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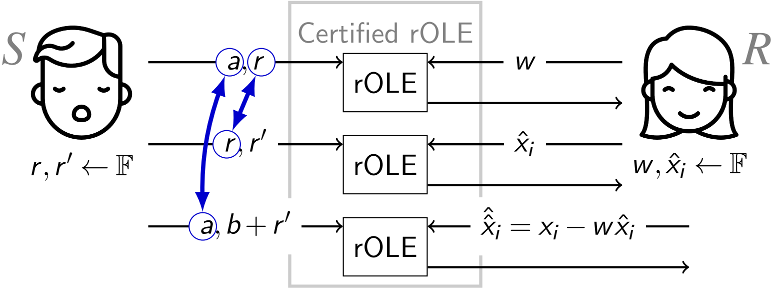
- "Strong" UC-secure against Sender:

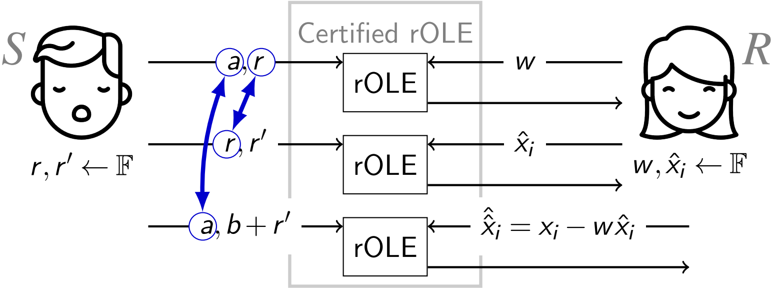
Certified rOLE



$$\underbrace{ax_i + b}_{\boxed{\text{Target}}} \;=\; \underbrace{(aw + r)}_{\boxed{\text{Commitment}(a)}} \cdot \hat{x}_i \;-\; \underbrace{(r\hat{x}_i + r') \;+\; e(a\hat{\hat{x}}_i + b + r')}_{\boxed{\text{rOLE outputs}}}$$

- Correctness: Above equation.
- UC-secure against Receiver: $x_i := w\hat{x}_i + \hat{\hat{x}}_i$.
- "Strong" UC-secure against Sender: Deviate $\implies$ random output

Certified rOLE

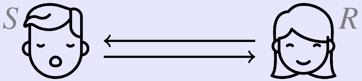


$$\underbrace{ax_i + b}_{\text{Target}} = \underbrace{(aw + r)}_{\text{Commitment}(a)} \cdot \hat{x}_i - \underbrace{(r\hat{x}_i + r') + e(a\hat{\hat{x}}_i + b + r')}_{\text{rOLE outputs}}$$

- Correctness: Above equation.
- UC-secure against Receiver: $x_i := w\hat{x}_i + \hat{\hat{x}}_i$.
- ~~"Strong"~~ UC-secure against Sender: Deviate $\implies$ random output
  not yet

Our Results

NEW primitive: Oblivious linear function evaluation (OLE)



$a, b \in \mathbb{F}$ $x \in \mathbb{F}$

get $ax + b \in \mathbb{F}$

Theorem 2

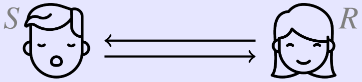
An information-theoretical UC-secure reusable NISC protocol in rOLE-hybrid model.

Theorem 3

An UC-secure 2-msg reusable OLE protocol in the CRS setting, under Paillier assumption.

Our Results

NEW primitive: Oblivious linear function evaluation (OLE)



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Theorem 2

An information-theoretical UC-secure reusable NISC protocol in rOLE-hybrid model.

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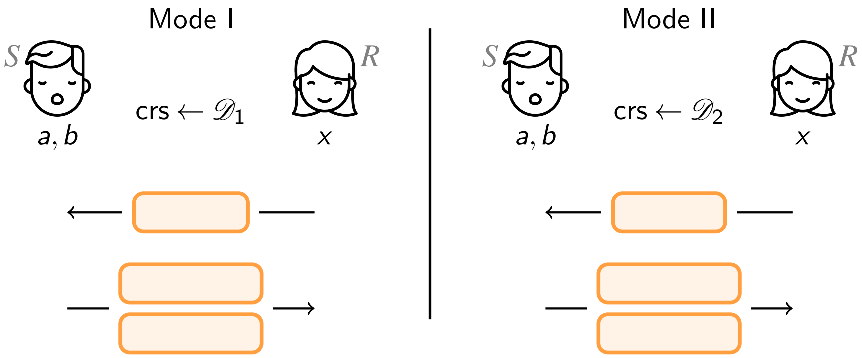
rOLE from Paillier

Dual-mode (similar to OT from [PVW'08])



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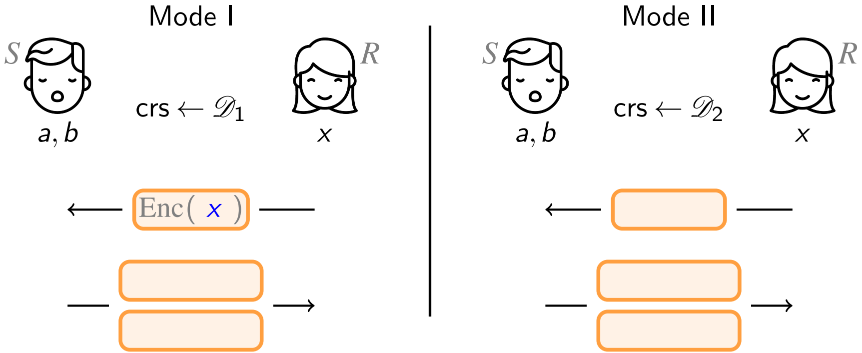

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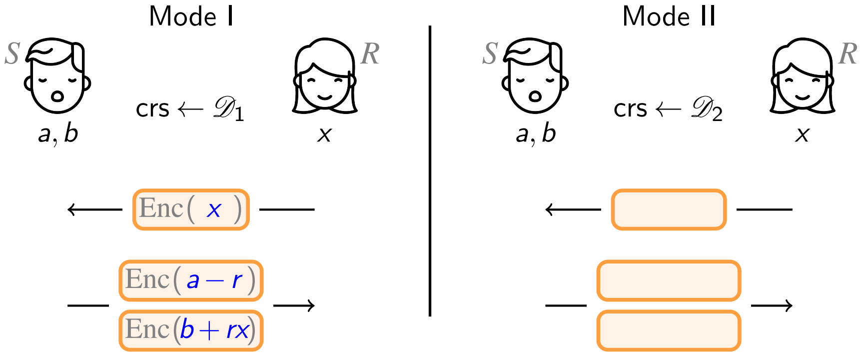

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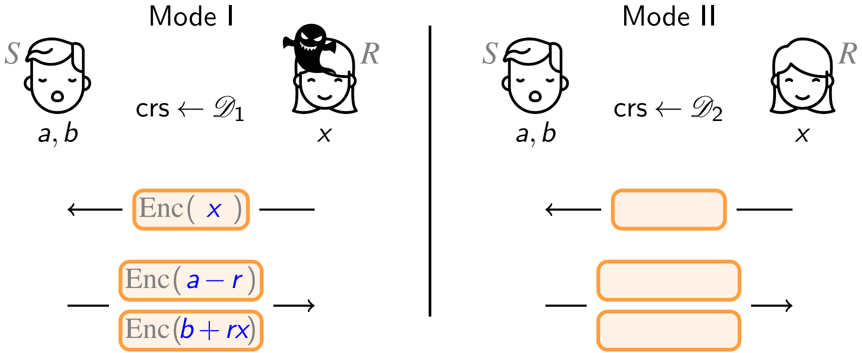
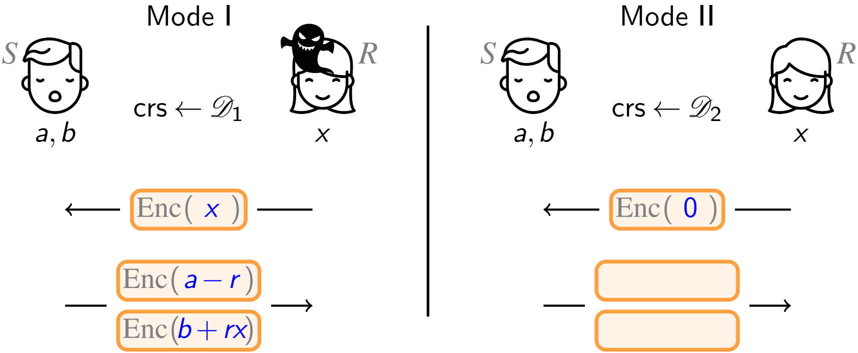


Efficient simulator against
unbounded malicious receiver

rOLE from Paillier

Dual-mode (similar to OT from [PVW'08])



Efficient simulator against
unbounded malicious receiver

rOLE from Paillier

Dual-mode (similar to OT from [PVW'08])

Mode I

S  a, b

$\mathsf{crs} \leftarrow \mathscr{D}_1$

 R x

$\xleftarrow{\quad} \boxed{\mathrm{Enc}(\ x\ )} \xleftarrow{\quad}$

$\xrightarrow{\quad} \boxed{\mathrm{Enc}(a - r)},\ \boxed{\mathrm{Enc}(b + rx)} \xrightarrow{\quad}$

Efficient simulator against
unbounded malicious receiver

Mode II

S  a, b

$\mathsf{crs} \leftarrow \mathscr{D}_2$

 R x

$\xleftarrow{\quad} \boxed{\mathrm{Enc}(\ 0\ )} \xleftarrow{\quad}$

$\xrightarrow{\quad} \boxed{\mathrm{Enc}(\ a\ )},\ \boxed{\mathrm{Enc}(\ b\ )} \xrightarrow{\quad}$

rOLE from Paillier

Dual-mode (similar to OT from [PVW'08])

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S

a, b

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$\xleftarrow{\quad \boxed{\mathrm{Enc}(\, x \,)} \quad}$

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Efficient simulator against unbounded malicious receiver

Mode II

S

a, b

$\mathsf{crs} \leftarrow \mathscr{D}_2$

R

x

$\xleftarrow{\quad \boxed{\mathrm{Enc}(\, 0 \,)} \quad}$

$\xrightarrow{\quad \boxed{\mathrm{Enc}(\, a \,)} \quad \boxed{\mathrm{Enc}(\, b \,)} \quad}$

Efficient simulator against unbounded malicious sender

rOLE from Paillier

Dual-mode (similar to OT from [PVW'08])

$\mathscr{D}_1$ is indistinguishable from $\mathscr{D}_2$

Mode I

S  R 

a, b $\qquad$ crs $\leftarrow \mathscr{D}_1$ $\qquad$ x

$\longleftarrow$ $\boxed{\mathrm{Enc}(\ x\ )}$ $\longleftarrow$

$\longrightarrow$ $\boxed{\mathrm{Enc}(a - r)}$, $\boxed{\mathrm{Enc}(b + rx)}$ $\longrightarrow$

Efficient simulator against unbounded malicious receiver

Mode II

S  R 

a, b $\qquad$ crs $\leftarrow \mathscr{D}_2$ $\qquad$ x

$\longleftarrow$ $\boxed{\mathrm{Enc}(\ 0\ )}$ $\longleftarrow$

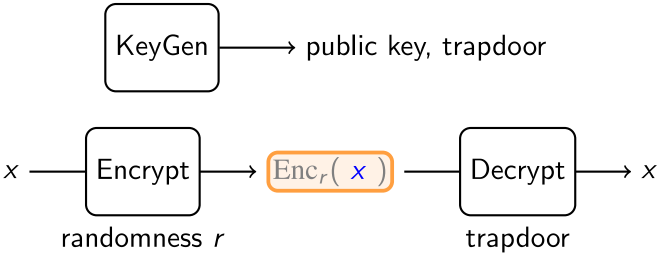
$\longrightarrow$ $\boxed{\mathrm{Enc}(\ a\ )}$, $\boxed{\mathrm{Enc}(\ b\ )}$ $\longrightarrow$

Efficient simulator against unbounded malicious sender

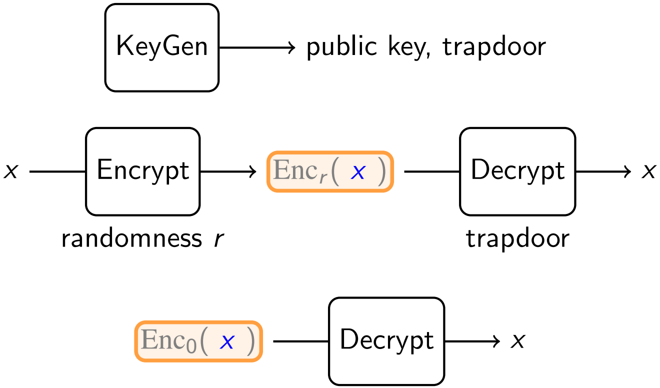
Paillier Encryption Scheme

$\boxed{\text{KeyGen}} \longrightarrow$ public key, trapdoor

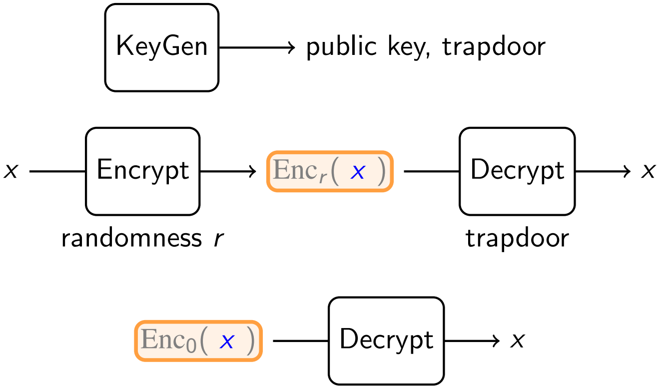
Paillier Encryption Scheme



Paillier Encryption Scheme

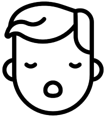


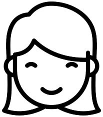
Paillier Encryption Scheme



$$\boxed{\mathrm{Enc}_r(\ x\ )} \cdot \boxed{\mathrm{Enc}_s(\ y\ )} = \boxed{\mathrm{Enc}_{r+s}(\ x+y\ )}$$

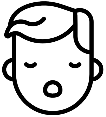
rOLE from Paillier

S

a, b

R

x

rOLE from Paillier

S



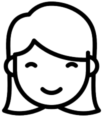
a, b

CRS (Mode I)

$$h = \boxed{\mathrm{Enc}_0(\ 1\ )}$$

$$w = \boxed{\mathrm{Enc}_\alpha(\ 0\ )}$$

$$W_0 = \boxed{\mathrm{Enc}_\beta(\ 1\ )}$$



R

x

rOLE from Paillier

CRS (Mode I)

$h = \boxed{\mathrm{Enc}_0(\ 1\ )}$

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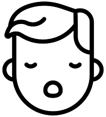
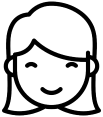
S



a, b

R



x

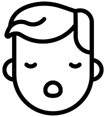
sample sk

$$\longleftarrow W_1 = w^{\mathsf{sk}} W_0^x = \boxed{\mathrm{Enc}_{x\beta+\alpha\cdot\mathsf{sk}}(\ x\ )} \text{———}$$

rOLE from Paillier

CRS (Mode I)

S



a, b

sample r

$$h = \boxed{\mathrm{Enc}_0(\ 1\ )}$$

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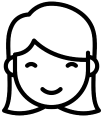
$$W_0 = \boxed{\mathrm{Enc}_\beta(\ 1\ )}$$

R



x

sample sk

$$\longleftarrow \quad W_1 = w^{\mathsf{sk}} W_0^x = \boxed{\mathrm{Enc}_{x\beta+\alpha\cdot\mathsf{sk}}(\ x\ )} \quad \text{———}$$

$$v = w^r = \boxed{\mathrm{Enc}_{r\alpha}(\ 0\ )}$$

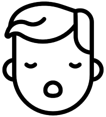
$$\text{———} \quad V_0 = h^a W_0^{-r} = \boxed{\mathrm{Enc}_{-r\beta}(\ a - r\ )} \quad \longrightarrow$$

$$V_1 = h^b W_1^r = \boxed{\mathrm{Enc}_{rx\beta+r\alpha\cdot\mathsf{sk}}(\ b + rx\ )}$$

rOLE from Paillier

CRS (Mode I)

S



a, b

sample r

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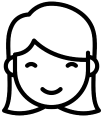
R



x

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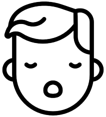
$$v^{\mathsf{sk}} V_0^x V_1 = \boxed{\mathrm{Enc}_0(\ ax + b\ )}$$

rOLE from Paillier

S



a, b

sample r

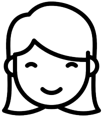
CRS (Mode II)

$$h = \boxed{\mathrm{Enc}_0(\ 1\ )}$$

$$w = \boxed{\mathrm{Enc}_\alpha(\ 0\ )}$$

$$W_0 = \boxed{\mathrm{Enc}_\beta(\ 0\ )}$$

R



x

sample sk

$$\longleftarrow\quad W_1 = w^{\mathsf{sk}} W_0^x = \boxed{\mathrm{Enc}_{x\beta+\alpha\cdot\mathsf{sk}}(\ x\ )}\quad\text{——}$$

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rOLE from Paillier

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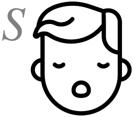

a, b

sample r

x

sample sk

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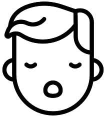
rOLE from Paillier

S



a, b

sample r

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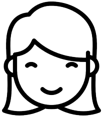
R



x

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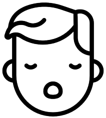
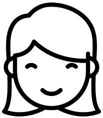


a, b

sample r

R



x

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"Strong" UC-security requires a machenism to detect malicious sender

Our Results

- **(!$\exists$ IT rNISC/rOT)** There is no information-theoretical reusable NISC protocol in rOT-hybrid model.
- **(IT rNISC/rOLE for arithmetic $\mathbf{NC}^1$)** Information-theoretical UC-secure reusable NISC protocol for any arithmetic $\mathbf{NC}^1$ circuit or arithmetic branching program in rOLE-hybrid model.
- **(IT rNIZK/rOLE)** Information-theoretical UC-secure reusable NIZK protocol in rOLE-hybrid model; $O(1)$ calls per gate.
- **Previous two + Garbled circuit $\rightarrow$ (rNISC/rOLE)** UC-secure reusable NISC for general circuits; IT secure against sender; poly(λ) calls per gate.
- **(rOLE protocol from Paillier)** UC-secure reusable 2-message OLE protocol in CRS model; one-side IT secure; c.c. $O(1)$ group elements per call.

Our Results

- **rNISC** in CRS model assuming the security of Paillier encryption.
- **rNIZK** in CRS model assuming the security of Paillier encryption. c.c. $O(1)$ group elements per gate.
- **Statistical designated-verifier NIZK argument** for NP in CRS model assuming Paillier.
- Push cryptograph to offline phase.
  In offline phase: prepare random $((a, b), (x, ax + b))$;
  In online phase: consume the prepared randomness.

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