

期 ≠ 11月8日

Pset 2 non-uniform

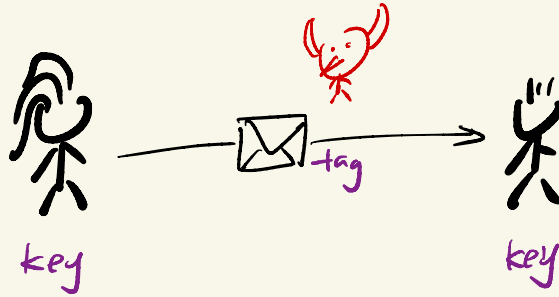
Pset 3  $\varepsilon_i \in \text{negl}(n) \not\Rightarrow \varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_n \in \text{negl}(n)$

$\exists i \varepsilon_i \notin \text{negl}(n) \not\Rightarrow \varepsilon_1 + \varepsilon_2 \notin \text{negl}(n)$

# Chosen-ciphertext attack (CCA)

- Secrecy
- Integrity

# message authentication code (MAC)



Previous:  
Passive Attack

Today:  
Active Attack

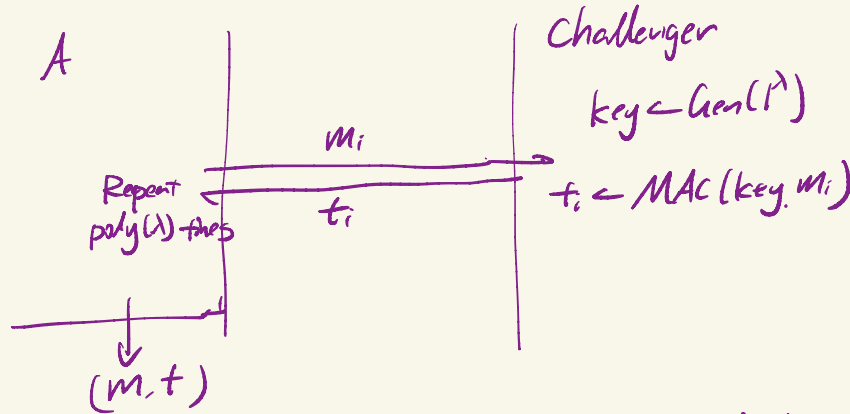
## MAC

- Key generation  $\text{Gen}(1^\lambda) \rightarrow \text{key}$
- Tag generation  $\text{MAC}(\text{key}, \text{msg}) \rightarrow \text{tag}$
- Verification  $\text{Verify}(\text{key}, \text{msg}, \text{tag}) \rightarrow \begin{cases} \text{Acc} \\ \text{Rej} \end{cases}$   
T F

## Correctness:

- $\text{key} \leftarrow \text{Gen}(1^\lambda)$
- $\text{tag} \leftarrow \text{MAC}(\text{key}, \text{msg})$
- $\text{Acc} \leftarrow \text{Verify}(\text{key}, \text{msg}, \text{tag})$

# MAC Forging Game $\text{MAC-forge}_{A, \pi}(\lambda)$



A wins if and only if

- (1)  $\text{Verify}(\text{key}, m, t) \rightarrow \text{Acc}$
- (2a)  $(m, t) \neq (m_i, t_i) \forall i$
- (2b)  $m \neq m_i \forall i$

$\pi$  is

- unforgeable if  $\forall \text{ p.p.t. } A \quad \Pr[\text{MAC-forge}_{A, \pi}(\lambda) \rightarrow 1] \leq \text{negl}(\lambda)$
- strongly unforgeable if  $\forall \text{ p.p.t. } A \quad \Pr[\text{MACSforge}_{A, \pi}(\lambda) \rightarrow 1] \leq \text{negl}(\lambda)$

MAC needs randomness?  
 NO, deterministic MAC.

$\Downarrow$   
 $\text{Verify}(k, m, t) = \text{check if } t = \text{MAC}(k, m)$

$\Downarrow$   
 unforgeability  $\equiv$  strongly unforgeability

Why No verification oracle?

# Chosen-Ciphertext Attack

$\text{PrivK}_{A, \Pi}^{\text{CCA1}}(\lambda)$

$\text{PrivK}_{A, \Pi}^{\text{CCA2}}(\lambda)$

P.P.t. A

Repeat  
 $\text{poly}(\lambda)$   
times

$m_i$

$c_i$

$c'_i$

$m'_i$

$m_0, m_1$

$c$

$m_i$

$c_i$

$c'_i \neq c_i$

$m'_i$

output  $b'$

Challenger

$\text{key} \leftarrow \text{Gen}(1^\lambda)$

$c_i = \text{Enc}(\text{key}, m_i)$

$m'_i = \text{Dec}(\text{key}, c'_i)$

$b \leftarrow \{0, 1\}$

$c = \text{Enc}(\text{key}, m_b)$

$c_i = \text{Enc}(\text{key}, m_i)$

$m'_i = \text{Dec}(\text{key}, c'_i)$

A wins iff  $b' = b$



CPA-secure encryption scheme + Unforgeable MAC  $\Rightarrow$  CCA1-secure

Enc ... + strongly unforgeable MAC  $\Rightarrow$  CCA2-secure  
Dec

$\Downarrow$  Authenticated Encryption  $\Uparrow$

Candidate (MAC-then-encrypt)

$$Enc'_{k,k}(m) = Enc_k(m, MAC_k(m))$$

$$Dec'_{k,k}(c) = m, t \leftarrow Dec_k(c)$$

Verify<sub>k</sub>(m, t) accepts  $\Rightarrow$  output m  
otherwise  $\Rightarrow$  output  $\perp$

Candidate (encrypt-then-MAC)

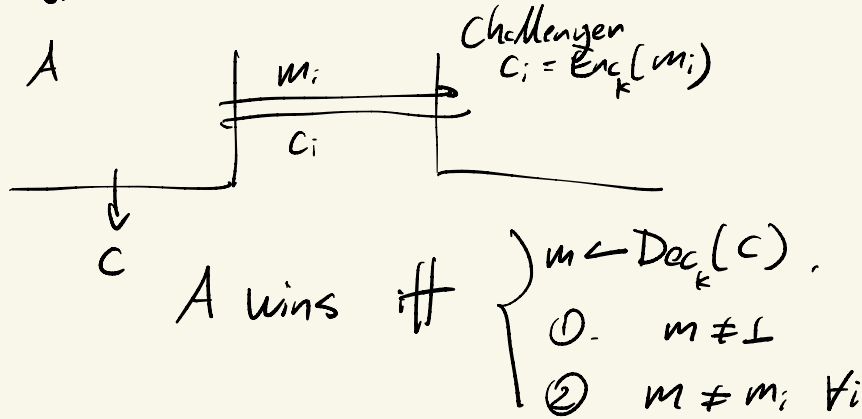
$$Enc'_{k,k}(m) = (c = Enc_k(m), t = MAC_k(c))$$

$$Dec'_{k,k}(c, t) = \text{Verify}_k(c, t) \text{ reject} \Rightarrow \text{output } \perp$$

otherwise  $\Rightarrow Dec_k(c)$

Def. Authenticated Encryption = CCA + Unforgeable

Encryption Unforgeability Game  $\text{Enc-Forge}_{A, \Pi}(\lambda)$



# Constructions of MAC

MAC for fixed-length messages.

assume PRF  $f: \{0,1\}^k \times \{0,1\}^n \rightarrow \{0,1\}^t$

◦ Gen( $1^k$ ) sample  $k \leftarrow \{0,1\}^k$

◦ MAC( $k, m$ ) =  $f_k(m)$

◦ Verify( $k, m, t$ ) =  $(f_k(m) = t)$

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◦ MAC( $k, (m_1, m_2, m_3, \dots, m_n)$ )

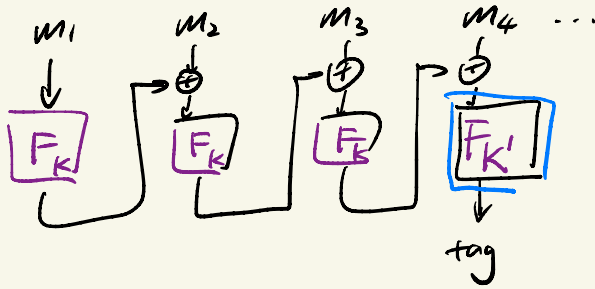
$(ZV, f_k(IV \parallel m_1 \parallel \dots \parallel m_n))$   
for  $i=1, \dots, n$

$m_1, m_2 \xrightarrow{\text{MAC}} t_1, t_2$

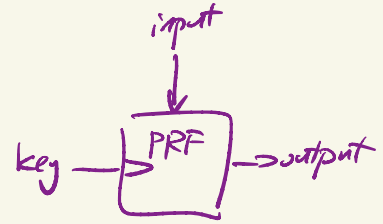
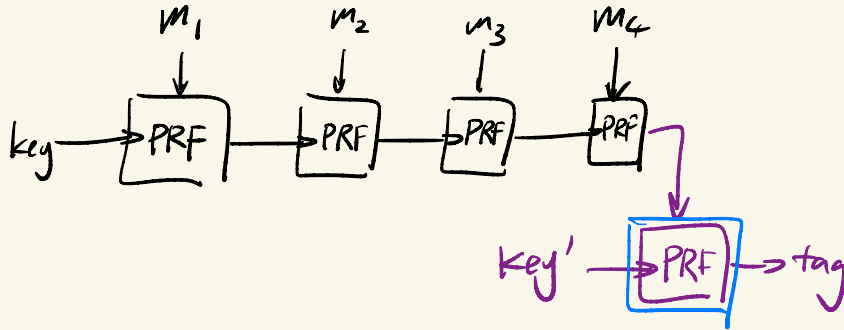
$m'_1, m'_2 \xrightarrow{\text{MAC}} t'_1, t'_2$

$m_1, m'_2 \xrightarrow{\text{MAC}} t_1, t'_2$

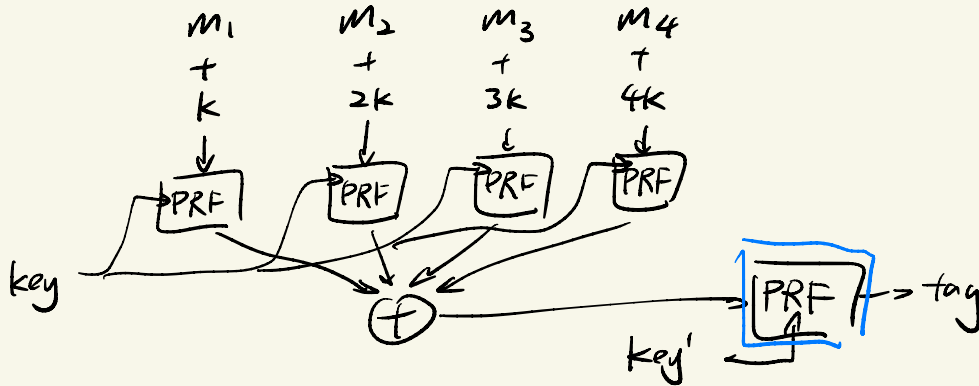
CBC-mode

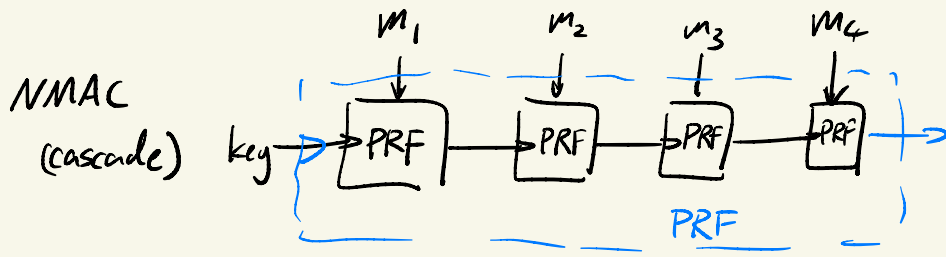


NMAC  
(cascade)

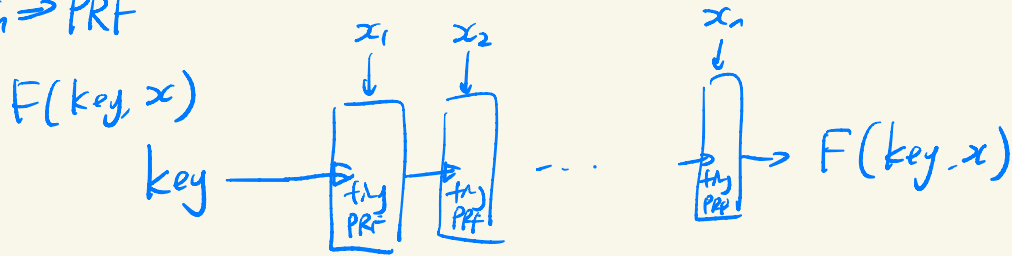


PMAC





PRG  $\Rightarrow$  PRF



$$\text{tiny PRF}: \{0,1\}^n \times \{0,1\} \rightarrow \{0,1\}^n$$

$$\text{key}, x \rightarrow \begin{cases} G(\text{key})[1:n] & \text{if } x=0 \\ G(\text{key})[n+1:2n] & \text{if } x=1 \end{cases}$$

## Prefix-free PRF

Prefix-free encoding:  $E: \{0,1\}^* \rightarrow \{0,1\}^*$

$\forall x_1, x_2, E(x_1)$  is not a prefix of  $E(x_2)$

$$\text{Im } E = E(\{0,1\}^*) = \{E(x) \mid x \in \{0,1\}^*\}$$

PRF for all strings  $\iff$  prefix-free encoding + prefix-free PRF

$$F^0: \{0,1\}^1 \times \{0,1\}^* \rightarrow \{0,1\}^1$$

$E$

$$F: \{0,1\}^1 \times \text{Im } E \rightarrow \{0,1\}^1$$

$$F^0(x) = F(\text{key}, E(x))$$



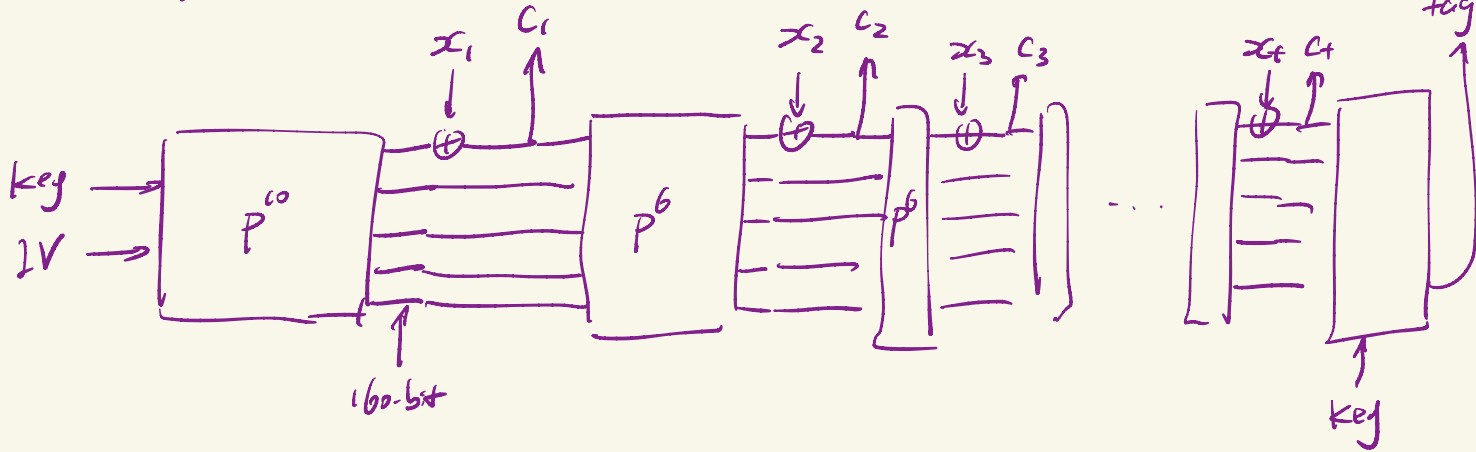
# Practical Authenticated Encryption

Eg. ASCON cipher

key

IV

$x_1, \dots, x_t \in \{0,1\}^{32}$



cipher<sub>text</sub> ( IV,  $c_1, \dots, c_t, \text{tag}$  )